THE ANOMALOUS TRANSPORT IN TURBULENT PLASMAS

Gy. Steinbrecher, N. Pometescu*, M. Negrea*, Dana Constantinescu**, Iulian Petrisor**

**Department of Physics, University of Craiova, Craiova*
***Department of Mathematics, University of Craiova, Craiova*

1. Particle and field line stochastic dynamics. Response to stochastic perturbations in tokamak

The problem of turbulent particle transport is still unsolved even for simple fluid models. In order to overcome the limitations related to the our incomplete mathematical understanding of the tokamak plasma turbulence, limitations related to the stability problems in numerical simulations, the concepts Self Organized Criticality (SOC) was proposed in [1], and partially justified analytically, by Random Linear Amplification model (RLA) in [2]. This work is intended to explore the possibility of first principle foundation, generalization, and application in the study of the anomalous particle transport, of the SOC and RLA models. The efforts are provided in the perspective of using the generalization of the SOC& RLA models, in order to elaborate "wind tunnel approach" for the optimization or design of next generation of tokamak.

The work from A1 and B1 is intended to extend the range of applicability the methods of the RLA model to more physical cases. Analytic result of A2 is useful in the interpretation of plasma turbulence data by SOC or RLA models. New, unexpected first principle argument in the favor of the SOC and RLA models follows from the numerical finding from A3. In B2 are exposed the numerical methods to be used in future development. In B3 two new experimental predictions of the RLA model are exposed, related to a new kind of anomalous transport in turbulent plasma and the dependence on the model parameters.

A. Elaboration of soluble model for the generation of the self similar, non-gaussian electrostatic fluctuations.

I. Gruzinov, P. H. Diamond, and M. N. Rosenbluth published pedestal width model based on SOC concepts in ref. [1], based on numerical simulations. Nevertheless, the applicability of SOC models in the explanation of plasma turbulence was questioned. Strong analytic support for SOC models of edge plasma turbulent transport and pedestal formation was given in the previous work on the RLA model [2]. The RLA model explains two different types of experimental data on plasma turbulence, obtained on DIII-D tokamak, presented in [3], [4].

The advantage of the RLA model is that the information on high intensity electric field fluctuations (encoded in the heavy tail exponent) can be obtained in a closed analytic form.

A1. Elaboration of soluble model for the generation of the self similar, non-gaussian electrostatic fluctuations.

Previous theoretical results [2] combined with experimental data [3], [4] give a strong support for the existence of small, Gaussian electrostatic or charge density fluctuations, triggering the onset of instabilities in the edge tokamak plasmas. A possible candidate mathematical model, supported by the above-mentioned works as well as by self-organized criticality models, applied in plasma turbulence, is the model of superdiffusive fractional Brownian motion (fBm). The linearized stochastic approximations of the Hasegawa –Mima and Hasegawa-Wakatani equations were studied, in order to obtain a soluble model for the generation of the self-similar non-Gaussian electrostatic fluctuations. The methods from ref. [2] were used. The linearization is performed by expansion around a stochastic background field that is a stochastic approximation of the exact solution. The stochastic background is represented by samples of fluctuating electric potential, modeled by fractional Brownian landscape respectively fractional Brownian cloud. The statistical properties of the linear part of the perturbations were studied.

- The asymptotic behavior of the linearized reduced MHD equations (the stochastic versions of the linearized Hasegawa –Mima and Hasegawa-Wakatani equations) was studied in analogy with the methods from ref. [2]. By analytical methods was proved that the asymptotic behavior of the solution of the associated probability distribution function of the electric field has a power law, self similar non-Gaussian asymptotic decay.
- In the limit of strong, increasing external magnetic field, we studied the anisotropy effect, in the case of stochastic linearized Hasegawa-Mima equation. By analytic methods was proved that the asymptotic self similarity exponent of the transverse component of the electric field fluctuations approaches zero and the asymptotic self similarity exponent of the parallel component of the electric field fluctuations remains finite
- In order to derive consequence, suitable for validation in Tokamak discharge experiments, the analogue functional Hopf equation, used for turbulence studies, was obtained, for the linearized stochastic approximation of the Hasegawa-Mima and Hasegawa-Wakatani equations. From the Hopf equations an upper bound on the rate of the instability growth for the case of Hasegawa-Mima equation was derived.

Conclusion: The linearized stochastic approximation of the Hasegawa-Wakatani and Hasegawa-Mima equations, in the strong magnetic field limit, provide a soluble model.

A2. Exactly soluble results on fractional Brownian motion [5].

In order to avoid the limitations related to the linear approximation used in previous models, the study of the instability triggering process, in the framework of the self-organized criticality models (SOC) was started. When the linear approximation of the instability growth fails, there is an alternative method to study the instability growth, triggered by random noises: it is possible to use arguments from SOC models used in tokamak physics (I. Gruzinov, P. H. Diamond, M. N. Rosenbluth, [1]). In such models it is important to know the statistics of the moments when a stochastic noise reaches a fixed instability threshold. In this end the problem of first exit time (FET) problem for physical models related to the fractional Brownian motion was studied by analytic methods.

The best-known result is of G. Molchan, (Commun. Math. Phys. **205,** 97-111, (1999)). The error term obtained, in the asymptotic expansion of the FET for large time T, is of the form $O(1/\sqrt{\log(t)})$. His result was improved. The new error term of the asymptotic expansion is bounded by $O(T^{H-1})$, where H is the Hurst (or self similarity) exponent of the fractional Brownian motion. These results can be used to extract the parameters (the self similarity or Hurst exponents) of the charge density fluctuations from the fluctuation of the flux measurements on the edge plasma. (These results also can be used to extract the solar model parameters from satellite measurements.)

A3. Numerical evidence for the generation of the stochastic time series like fractional Brownian motion.

Numerical evidence was obtained for the fact that deterministic, independent, uncorrelated particle motion according to the equations of the chaotic field line dynamics corresponding to the ergodic domains produces electric noise whose time behavior can be approximated by superdiffusive fractional Brownian motion. The self similarity exponent (the Hurst exponent) has different values for radial and poloidal components: the Hurst exponent of the radial components is close to 0.8, while the exponent of the poloidal component is around 0.52. This difference in the Hurst exponents is the direct consequence of the geometry of the magnetic field lines in tokamak. These results give a new theoretical support for the model from [2] of the edge plasma turbulence in tokamak.

Conclusions:

- First principle model for the generation of non-gaussian electrostatic perturbation was obtained. From this model can be obtained exactly soluble predictions on the anomalous particle transport in tokamak in the linearized approximation.
- Exact result in non-linear models, concerning the statistics of nonlinear instability triggering effects was obtained.

B. Elaboration of models described by stochastic differential equations.

B1. Extension of model ref. [2], by removing the assumption on the over damped approximation.

The validity of the key result of ref. [2] model was proved in a more physical case, when the assumption that the system under study is in the over damped regime is removed. The formula for heavy tail exponent was generalized. (*B. Weyssow,* ***G. Steinbrecher,*** General Linear Stochastic Model for Edge Plasma Turbulence, presented by B. Weyssow, at JET Task Force Meeting, 11 May 2004, and ***G. Steinbrecher****, B. Weyssow,* Solvable Self similar Model of Plasma Turbulence, presented at Trilateral Euregio Cluster (TEC), F.O.M. Rijnhuizen, 11 May 2004.) This result gives a new argument for modelling the anomalous transport in tokamak by self-organized criticality models.

B2. Elaboration of improved numerical methods and statistical extrapolations for solution of stochastic differential equations.

The mathematical foundation of stochastic integration methods for the numerical solution of the stochastic linearized Hasegawa – Mima and Hasegawa-Wakatani equations was elaborated, by generalization to the case of stochastic linear partial differential equations, of the stochastic version of the Euler method, known in one-dimensional case. The Richardson extrapolation methods, used in the numerical integration of the deterministic differential equations, were extended to the case of stochastic partial linear differential equations. The extrapolation methods reduce the CPU time in simulations. Optimized C++ programs [6] for generation of the sample path of the following processes were elaborated:

a) General Gaussian stationary process, with a given, arbitrary, time correlation.

b) By using the previous program, for a particular choice of the correlations, the paths of fractional Brownian motion was generated.

c) Program for the generation of two and three-dimensional analogue of the (one dimensional, classical) fractional Brownian motion: fractional Brownian landscape in two and fractional Brownian cloud, in three dimensions, was written.

d) By using the program from c), a C++ program was elaborated, which solves the stochastic linearized approximation of the Hasegawa-Mima equation.

e) Numerical tests concerning the charged particle motion were performed. The test particle is exposed to electrostatic, random field generated by random charge distribution, modeled by fractional Brownian landscape. In the over damped approximation we tested our numerical results with the one-dimensional, soluble, Landau model, (of particle motion in a random force field in over damped approximation).

B3. Response to perturbations: numerical and analytical study of the particle dynamics in the tokamak background field perturbed by fluctuations with self similar and non-Gaussian statistics.

- The effect of the correlation of the additive and multiplicative random terms of the linear stochastic processes, on the particle motion.

Concerning the existence and stability of the stationary state of the ref. [2] model random linear system was proved that the stationary distribution function exists when the additive and multiplicative noise are non-correlated. When the correlation is positive, then the maximum of the stationary probability density function is shifted to negative values, respective to positive values in the case of anti-correlation. When the non-correlated part of the noises is very small, then the population of the heavy tail, as well as the cross-field transport is drastically reduced. Nevertheless, the cross-field transport is more intense compared to the usual superdiffusive transport mechanisms: the higher moments of the velocity diverges exponentially. (*B. Weyssow,* ***G. Steinbrecher****,* General Linear Stochastic Model for Edge Plasma Turbulence, presented by B. Weyssow, at JET Task Force Meeting, 11 May 2004, and ***G. Steinbrecher****, B. Weyssow,* Solvable Self similar Model of Plasma Turbulence, presented at Trilateral Euregio Cluster (TEC), F.O.M. Rijnhuizen, 11 May 2004.)

This preliminary result can give a hint on the possibility of reducing the particle transport in tokamak.

- Transverse and longitudinal velocity probability density function (PDF) of particles in tokamak.

Results are derived in the conditions of previous model from ref. [2]. By analytic methods was proved that the transverse velocity probability density function has a heavy tail and the transverse fluctuating electric field has the same heavy tail exponent. The longitudinal stationary particle velocity distribution, when exists, has the heavy tail exponent equal with the exponent of the PDF of the longitudinal electric fluctuations, in the collisional case.

In the idealized, exact asymptotic self-similar case, the stationary velocity probability density function does not exist: the large time limit of the probability density function, according to the RLA model, is zero. In real cases, when there is an upper cutoff of the correlation times, the transversal velocity probability density function decreases like the inverse of the transversal (to magnetic field) velocity, in the domain of the approximate self-similarity. It is followed by an exponential fall-off. The longitudinal velocity probability density function has the same behavior, in the collisional case. The statistical properties of the particle motion, derived from previous velocity probability density function, are strongly anomalous. The associated particle loss from tokamak is larger compared to the conventional, algebraic, super diffusive transport.

Conclusion:

New generalizations of previous tokamak anomalous transport models, based on stochastic differential equations were elaborated and the consequences on particle transport were studied. New C++ programs for further studies were written.

2. <u>Effects of the radio-frequency heating on the edge plasma transport</u>

The variation of the particle and energy fluxes with the absorbed radio frequency heating power is analysed for both turbulence and non-turbulence driven processes.

A. Radial and poloidal particle and energy flux of the ions (majority species) in turbulent plasmas in the presence of the ICRH.

The correlated influences of turbulence and radio-frequency heating on the radial transport in fusion plasma are investigated from a theoretical point of view. The problem was considered in a tokamak with toroidal axis symmetry magnetic field geometry and the equilibrium distribution function slightly different from Maxwell distribution function. The variation of the transport coefficients with radio-frequency power density, measured in terms of the Stix parameter, was analytically described in a theoretical model with four free parameters. We have plotted numerically the transport coefficients relative to Stix parameter for given values (relevant to ITER conditions) of the other three parameters [7]. The results were partially obtained in collaboration with B. Weyssow from Universite Libre de Bruxelles and J. Misguisch from C.E.A. Cadarache.

B. Radial particle and energy flux of the ions (majority species) in non-turbulent plasmas in the presence of the ICRH.

The radial energy flux driven by collisions and the linear radio frequency operator were evaluated in magnetically confined toroidal plasma. With $\widetilde{F}_0^i$ representing the leading order of the equilibrium ion particle distribution function depending on gyro phase angle, the collision operator is written as $C\left(\widetilde{F}_0^i\right)= C_{PAS}\left(\widetilde{F}_0^i\right)+ v_{\|} N_C^i \widetilde{F}_0^i$ and similar, the linear radio frequency (rf) operator, $Q\left(\widetilde{F}_0^i\right)= Q_{PAS}\left(\widetilde{F}_0^i\right)+ v_{\|} N_Q^i \widetilde{F}_0^i$. Here $v_{\|}$ is the parallel particle velocity and N_C^i is the remaining no pitch-angle-scattering part of collision operator. The contributions were obtained only from pitch angle scattering part. In the case of a reference distribution function slightly different from a Maxwellian and with an axis symmetry toroidal magnetic field geometry, the radial ion-particle flux $\Gamma_C^{i,rad}$ and energy flux $W_C^{i,rad}$ driven by collisions was obtained in the form (where ν_i is the collision frequency of the ions, n_i the ion-particle number density, T_e the electron temperature):

$$\left\langle \Gamma_C^{i,rad} \right\rangle_S = -\nu_i A_C \left[\gamma_{C,n} \frac{d \ln n_i}{dr} + \gamma_{C,T} \frac{d \ln T_e}{dr} \right]$$

$$\left\langle W_C^{i,rad} \right\rangle_S = -\nu_i D_C \left[w_{C,n} \frac{d \ln n_i}{dr} + w_{C,T} \frac{d \ln T_e}{dr} \right]$$

where $\langle \; \rangle_S$ represents the surface average. Here are given only terms provided by radial variation of particle density and temperature. The coefficients A_C and D_C depends both on the ion particle parameters and the Stix parameter but the coefficients $\gamma_{C,n}, \gamma_{C,T}, w_{C,n}$ and $w_{C,T}$ depends on the Stix parameter and integrals on the dynamical variables which was numerical evaluated. The radial particle $\Gamma_Q^{i,rad}$ and energy fluxes $W_Q^{i,rad}$ driven by rf heating are given as

$$\left\langle \Gamma_Q^{i,rad} \right\rangle_S = \frac{D_\perp}{k_\|} A_Q \left[\gamma_{Q,n} \frac{d \ln n_i}{dr} + \gamma_{Q,T} \frac{d \ln T_e}{dr} \right]$$

$$\left\langle W_Q^{i,rad} \right\rangle_S = \frac{D_\perp}{k_\|} D_Q \left[w_{Q,n} \frac{d \ln n_i}{dr} + w_{Q,T} \frac{d \ln T_e}{dr} \right]$$

Here $D_\perp$ the rf quasilinear diffusion coefficient, $k_\|$ parallel wave-number of the rf wave, n_i the ion particle number density and T_e the electron temperature. The coefficients A_Q and D_Q depends on the resonant ion Larmor frequency, ion particle parameters and the

Stix parameter. The coefficients $\gamma_{j,k}, w_{j,k}$ (j=C,Q; k=n,T) are evaluated explicitly [8] as functions of parameter Stix ξ.

3. <u>The study of the influence of the trapping on the diffusion of a quasi-particle (drift wave packets) in turbulent plasma using the decorrelation trajectory method</u>.

This study is devoted to the calculus of the Lagrangian correlation tensor and the diffusion tensor (for a plasma in a state of drift wave turbulence) for a large numbers of different parameters involved in the numerical analysis. A complete study of the influences of the parameters was done. The diffusion tensor and the Lagrangian correlation tensor depend on the following parameters: the initial wave vector ($\mathbf{k_0}$), the two Kubo numbers K and K_d and the anisotropy parameter Λ. The specific Eulerian correlations introduce the characteristic correlation lengths, λ_x, λ_y and the correlation time τ_c measuring the extent (in space and time) of the effectively correlated zone. With ε (the amplitude of the electrostatic fluctuations), λ_x, λ_y and the correlation time τ_c we can construct the electrostatic Kubo number K, defined as (in our model): $K = \varepsilon c \tau_c/(B_0 \lambda_x \lambda_y)$ where, B_0 is the intensity of the main magnetic field. In a similar way we define the diamagnetic Kubo number K_d as: $K_d = \tau_c V_{ds} / \lambda_x$ where V_{ds} is the ion acoustic diamagnetic drift velocity; the anisotropy parameter Λ is defined as the ratio $\Lambda = \lambda_x/\lambda_y$. The zonal flow structures exist if the ratio R satisfy the condition: $R = D^{as}_{kx|kx} / D^{as}_{ky|ky} > 1$, where $D^{as}_{kx|kx}$ and $D^{as}_{ky|ky}$ are the asymptotic diagonal diffusion coefficients in our model. It was considered in our analysis the following ranges for the parameters: Λ in the range [0.1, 1]; K in the range [0.01, 6]; two values for K_d, K_d = {0.5; 1} and 12 different initial wave vectors. The total numbers of sets (more than 300) of Lagrangian correlation tensors, running diffusion coefficients and asymptotic diffusion coefficients were calculated.

It was obtained enough information in order to conclude the following:

- *The value of the initial wave vector* $\mathbf{k_0}$ is important for the generation of the zonal flow structures.

  The following different cases were observed from our numerical analysis:

  - For $r = (k_{0x} / k_{0y}) < 1$, the zonal flow (R>1) is generated for any value of Λ.
  - For r>1, the zonal flow (R>1) appears for $\Lambda \le 0.6$.
  - The influence of the initial wave vector (i.e. the influence of the ratio r) disappears for $\Lambda \le 0.2$; for each value of $\Lambda \le 0.2$ practically the same value of R is obtained for any value of r.
- *The Kubo number* K is very important because it is related to the turbulent state. It was obtained for any initial wave vector and for any fixed values of Λ two regimes: for weak turbulence (K<1) in the log-log representation, the quasilinear regime (this regime corresponds to the weak turbulence state, i.e., K<1, and in this case the diffusion coefficient scaling as: $D \approx K^2$) is recovered (the slope = 2) and in the relatively strong turbulence case (K > 1) an almost flat regime with the slope approximately 0.1 is obtained.
- *The anisotropy parameter* Λ has also a strong influence for the generation of the zonal flow structures for Λ < 0.2. Increasing the anisotropy parameter (e.g. for isotropic case Λ=1)

the influence of the initial wave vector becomes dominant. The diagonal asymptotic diffusion coefficients are affected in very different way. For instance, in $\log_{10}(D_{jj}^{as})$ versus $\log_{10}(K)$ plot, as Λ varies (from 1 to 0.1) $\log_{10}D^{as}_{kx|kx}$ moves *upwards* while $\log_{10}D^{as}_{ky|ky}$ moves *downwards*. In Figure 1 the asymptotic diagonal diffusion coefficients as functions of K for four values of Λ [Λ = 1 (solid), Λ = 0.4 (dotted), Λ = 0.2 (dash-dotted), Λ = 0.1 (dashed)] and K_d=1; log10 − log10 representation for the initial wave vector ($k_{0x} = \sqrt{0.2}$, $k_{0y} = \sqrt{0.8}$) are represented.

- *The diamagnetic Kubo number* has a weak dependence on the zonal flow generation. Until now, we have not extended the analysis for this parameter (only two values were considered).

As a main result of this research it can be stated that in an anisotropic state of the turbulence (e.g., given by Λ=0.1), the asymptotic diffusion coefficient $D^{as}_{kx|kx}$ can become as much as 250 times larger than $D^{as}_{ky|ky}$ for any initial wave vector (see Figure 2). This produces an evident fragmentation of structures in x- (radial) direction and their elongation in the y- (poloidal) direction.

The important results of research were submitted to publication [9].

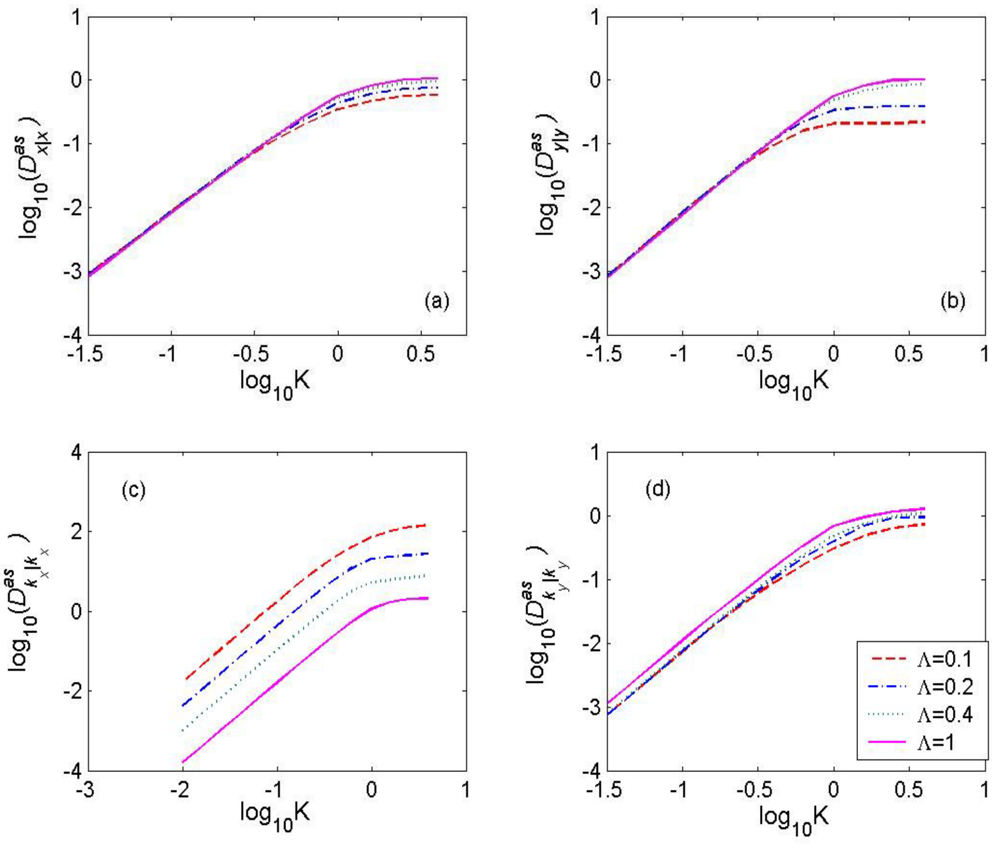

Figure 1: The asymptotic diagonal diffusion coefficients as functions of K for four values of Λ [Λ = 1 (solid), Λ = 0.4 (dotted), Λ = 0.2 (dash-dotted), Λ = 0.1 (dashed)]

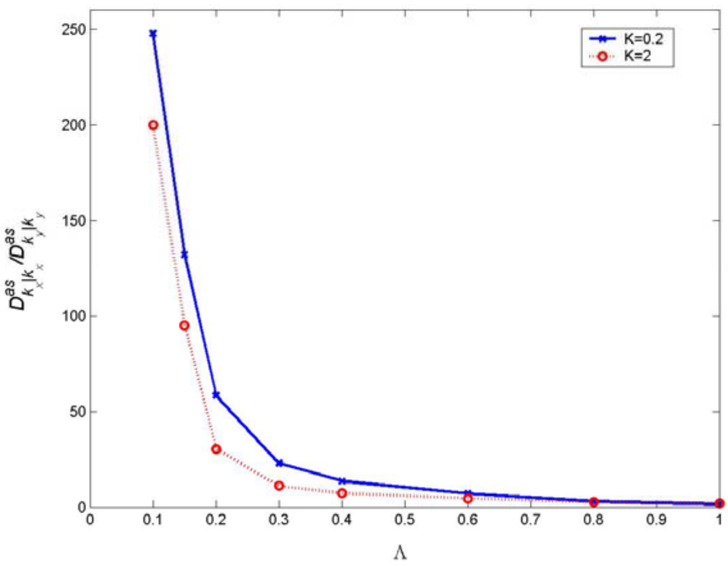

Figure 2: The ratio $D^{as}_{kx|kx}$ / $D^{as}_{ky|ky}$ as a function of Λ, for two values of the electrostatic Kubo number: K = 0.2, K = 2 (for the initial wave vector ($k_{0x} = \sqrt{0.2}$, $k_{0y} = \sqrt{0.8}$)) and K_d=1

4. The transport of collisionless thermal electron in a sheared unperturbed magnetic field and a fluctuating electric field using the decorrelation trajectory method.

In this study it was analyzed the diffusion of the collisionless thermal electrons in a combined fluctuating 2-dimensional electrostatic field and a sheared unperturbed confining magnetic field. The model is similar on a Langevin type system of equations that describes the motion of a guiding center through the highest significant order in the drift parameter. It was used in our study the *decorrelation trajectory method*; the combined effect of the shear and the electrostatic fluctuations, i.e., the competition between the shear Kubo number K_s and the electrostatic Kubo number K on the trapping regime was observed. The electrostatic Kubo number is defined as $K=\varepsilon c\tau_c/(B_0\lambda^2_\perp)$ where ε is the amplitude of the electrostatic fluctuations, τ_c is the correlation time, B_0 is the intensity of the main magnetic field and $\lambda_\perp$ is the correlation length perpendicular to the main magnetic field. The shear Kubo number K_s is defined as: $K_s = V_{th}\,\tau_c / L_s$ where V_{th} is the thermal velocity of the electron and L_s is the shear length. The main contributions of our work consisted in the calculus of the running and asymptotic radial and poloidal diffusion coefficients for different ranges of the electrostatic and shear Kubo numbers. The radial diffusion is diminished by an increase of the shear Kubo number for a fixed electrostatic Kubo number. The poloidal diffusion coefficient increases in a relatively strong turbulence regime when the shear Kubo number increases. A scaling law for the quasilinear turbulence regime was obtained for both diffusion coefficients. The influence of the magnetic shear on the diffusion coefficients of a test-electron moving in an electrostatic fluctuating field was not analyzed in both small and high electrostatic Kubo number regimes using the *decorrelation trajectory method* [10]. We intend in a future paper to study by a numerical simulation the thermal electron diffusion and compare the results obtained by these two methods.

5. <u>The study of the diffusion of a stochastic anisotropic sheared magnetic field lines using the decorrelation trajectory method.</u>

There are few papers that studied by direct numerical simulation the diffusion coefficient of the magnetic field lines of an *unshared* stochastic magnetic field in the case that all the turbulence correlation lengths λ_x , λ_y and λ_z are different [9], [11]. In our work it was developed the theoretical framework necessary to study the diffusion of the magnetic field lines for a stochastic *anisotropic sheared* magnetic field using the decorrelation trajectory method. This study is important for a future analysis concerning the diffusion coefficient of a particle moving in such a sheared stochastic magnetic field; there are a lot of papers devoted to the study of diffusion of a *particle* moving in a sheared stochastic electromagnetic field (see, e.g. [12]). The starting point of this study is the paper [13] where was analyzed the diffusion of a sheared magnetic field lines in the case of the *isotropic* turbulence. Until now, it was obtained analytically the Lagrangian correlations tensor, the running diffusion tensor and the asymptotic diffusion coefficient tensor for the anisotropic sheared stochastic magnetic field in the slab geometry approximation. It was also analyzed the decorrelation trajectories in various subensembles. Because there is expected a competition between the three parameters entering the theory (the magnetic Kubo number $K_m=\beta\lambda_z/\lambda_x$, the shear parameter $\theta_s = \lambda_z/L_s$ and the anisotropy parameter $\Lambda = \lambda_x/\lambda_y$.), a carefully analysis must be done in order to take into consideration only the physically possible regimes; here β is a measure of the magnetic field fluctuation, λ_z is the correlation length parallel to the main (unperturbed) magnetic field B_0 and L_s is the shear length. Until now the following values of the parameters were used: for K_m the range $[10^{-2}, 1]$ and the value $K_m=2$, for the shear parameter $\theta_s=\{0, 0.1, 0.5, 1, 2, 3\}$ and for the anisotropy parameter $\Lambda=\{0.2, 0.5, 1\}$. At least locally, anisotropy of the stochastic field is possible to exist and different diffusion coefficient in the radial and poloidal direction (the two directions perpendicular to the main magnetic field) can be calculated. For instance, in the case of the radial diffusion coefficient, for a fixed value of K_m and Λ the increase of shear produces a decrease of the diffusion coefficient (an important enhancement of the trapping effect). A more decrease of the diffusion is obtained if Λ becomes smaller than $\Lambda=1$ (the isotropic case) for fixed values for K_m and θ_s. The goal is to find (and to explain) the physical situations (anomalous transport regimes) when a massive decrease of the diffusion is observed (this kind of result is important for the fusion program).

Partial results were already published [14]. The partial analysis of the diffusion tensor shows that the so-called quasilinear regime (small values for K_m, i.e. the range $(10^{-2}, 1)$ is influenced by the anisotropy parameter Λ. For $\Lambda=1$ the standard quasilinear regime (the characteristic exponent p=2, $D^{as} \sim (K_m)^p$) is recovered; for $\Lambda<1$ the characteristic exponent for the quasilinear regime becomes <2. For each value of Λ, and for different values of θ_s the domain of the quasilinear regime seems to be "shorted", e.g. the quasilinear regime (the characteristic exponent p=2) is recovered in the range $K_m<10^{-1}$.

The complete numerical analysis necessary to calculate the asymptotic diffusion coefficients is still in progress.

6. Scenarios with Internal Transport Barriers (ITB).

For the confinement of plasmas by toroidal magnetic fields it is essential that the magnetic field line differential equations be as close as possible to the integrable case (in this case the magnetic field lines lie approximately on nested toroidal flux surfaces). This is because, to lowest order in Larmor radius, particle orbits are tied to magnetic field lines and particle trajectories may be described using straight-field –line flux coordinate.

Even if the magnetic field is not completely integrable some zones characterized by a reduced magnetic transport, usually surrounded by chaotic zones can be identified (for example in a reversed shear configuration in tokamaks). They correspond to the internal transport barriers (ITB) observed in experiments (where they are characterized by a jump in the slope of density and temperature profiles and a strong anomalous transport inside).

The magnetic internal transport barriers are zones where the magnetic field is almost integrable. They separate the plasma core region from the peripheral zone of the tokamak. The existence of a transport barrier ensures the confinement of the magnetic field's lines starting from the core region because the magnetic transport through the transport barrier is suppressed; hence the transport barrier prevents the outside radial motion of charged particles. For this reason the study of the transport barriers is extremely important.

We focused on the study of the magnetic internal transport barriers in reversed magnetic shear configuration in tokamak using the theory of dynamical systems and a Hamiltonian model (the rev-tokamap model) proposed in 1998 by R. Balescu (Phys. Rev. E 3, 3782 (1998)).

In the following presentation (r,θ,ζ) are the toroidal coordinates, $\psi = r^2/2$ is the toroidal flux, θ is the poloidal angle and ζ is the toroidal angle. The Poincare surface of section is a poloidal section $S:\zeta = cst$. The intersection of a magnetic field line starting from the point (θ,ψ) of S with S after a toroidal turn is denoted by $(\overline{\theta},\overline{\psi})$. The map that describes the dynamics of the magnetic field lines has the general form

$$T_k : \overline{\psi} = \psi - kg'(\theta)h(\overline{\psi}),\quad \overline{\theta} = \left(\theta + W(\overline{\psi}) + kg(\theta)h'(\overline{\psi})\right)(\text{mod}1).$$

The rev-tokamap is obtained using the quadratic winding function $W(\psi) = w\left(1 - A(C\psi - 1)^2\right)$ and $g(\theta) = -\dfrac{1}{4\pi^2\cos(2\pi\theta)}$, $h(\psi) = \dfrac{\psi}{1+\psi}$. The winding function has the maximum value w and the other parameters involved in its definition are $A = \left(w - W(0)\right)/w$, $C = 1 + \sqrt{\left(w - W(1)\right)/\left(w - W(0)\right)}$. The model describes a reversed shear configuration because the safety factor $q(\psi) = \dfrac{1}{W(\psi)}$ has a local minimum. The stochasticity parameter k is related to the amplitude of the perturbation of the integrable case (corresponding to $k = 0$).

An internal transport barrier that surrounds the magnetic line having the maximum rotation number (the shearless curve) was identified and described in the previous work [15] and an analytical explanation for the destruction of closed magnetic surfaces was proposed in [16].

The specific new topics of the research in the reported period are:

a) to realize a study of the dynamics of the magnetic field lines inside the transport barrier (in the low magnetic shear region of the tokamak) in order to explain the variation of its width and of its position for various q-profiles and perturbations.

b) to explain the effect of the main rational surfaces of the size of ITB in reversed shear models

c) to apply some methods of control theory in order to increase the size of an existing transport barrier or to obtain such a barrier if it does not exist in the uncontrolled system.

The main results are the following:

a) A decreasing sequence of invariant annuli, called non-twist annuli of order n and denoted by NTA_n, with regular dynamics was analytically obtained inside ITB [17]. The intersections of NTA, NTA_1, NTA_2 and NTA_3 with the line $\theta = 0.5$ are presented in Figure 3. The shearless curve is contained in every non-twist annulus.

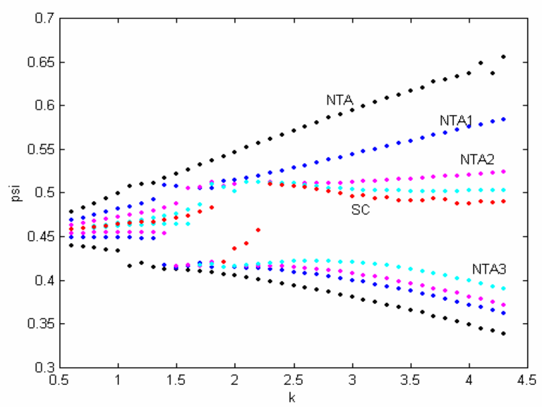

Figure 3 The intersection of the non-twist annuli with the line $\theta = 0.5$ *in the revtokamap model corresponding to a fixed winding function (* $w = 0.67$*,* $W(0) = 0.3333$*,* $W(1) = 0.1667$ *and various values of* $k \in [0.5, 4.5]$

This sequence can be used for localizing the shearless curve and it can be used for determining analytically zones with a reduced transport of particles for large perturbations. This sequence is also a useful tool for determining the amplitude of perturbations for which the transport barrier still exists. Their properties were related with the reconnection phenomena previously analyzed: when the reconnection occurs the twin island chains involved in reconnection are contained in all non-twist annuli [18].

b) Analytical explanations for some experimental observations concerning magnetic ITB in were deduced from the analytical properties of the models. The transport barriers are generally obtained in the low magnetic shear region.

The occurrence of the barrier is due to the fact that the system is almost integrable in an annulus situated in the region of low magnetic shear, being very close to a rigid rotation. The reversed shear of the magnetic field is not a mandatory condition for the presence of the ITB in the low shear region (a system having a monotonous q-profile and a large ITB, namely the degenerate tokamap, was proposed and studied) [19].

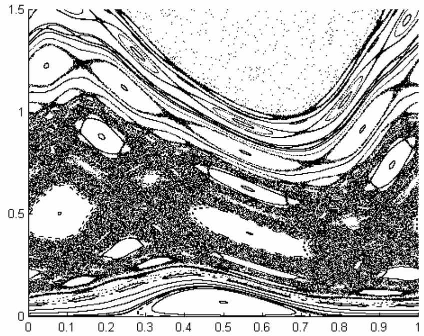

Figure 4 The phase portrait of the tokamap ($k = 5$)

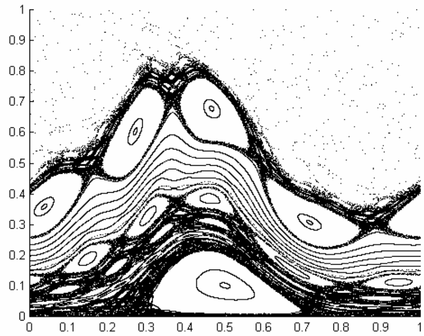

Figure 5 The phase portrait of the degenerate tokamap ($k = 5$)

The winding function of the tokamap is $W_T(\psi) = (2-\psi)(2-2\psi+2\psi^2)/4$. It is monotonous and $W'(\psi) \neq 0$ for all ψ. The winding function of the degenerate tokamap is $W_{DT}(\psi) = W_T(\psi) + \psi^2(2-7\psi/4)$. It is monotonous but the equation $W'(\psi) = 0$ has real solutions. In Figure 4 is presented the phase portrait of the tokamap (with a monotonous q-profile) and in Figure 5 is presented a phase portrait of the degenerate tokamap, both corresponding to k=5. A large transport barrier is formed in the degenerate-tokamap system..

In reversed shear tokamaks a zone with reduced transport can be noticed near the points where the q-profile has low order rational values.

The enlargement of the magnetic transport barrier is due to the fact that the twin island chains involved in the reconnection process enter the non-twist annulus and the magnetic transport barrier is enlarged [19]. This phenomenon has influence on the particle transport and it is observed in the experiments. It is a typical non-twist phenomenon and it does not occur for a monotonous q-profile, even if a large transport barrier exists.

c) A systematic study on the influence of the safety factor and the perturbation's amplitude on the position and on the size of the internal transport barrier (ITB) in the rev-tokamap model was realized in order to choose an appropriate control strategy [20]. The strategy of the usual control techniques in Hamiltonian systems is to modify the perturbation of the integrable system. Numerical simulations show that small modifications of the safety factor have important consequences on the ITB's size [19]; hence the control factor has to target the safety factor. From an experimental point of view the modification of the safety factor is realizable, even easier than to control the (mathematical) perturbations. For this reason a specific (mathematical) control technique was proposed. Taking into account that the transport barrier is localized in the low magnetic shear, a control term may be added to a monotonous q-profile in order to induce a low (zero) magnetic shear. In the controlled system the transport barrier exists and, by consequence, the magnetic confinement is realized. An example of controlled system originated from the tokamap model is the degenerate tokamap (previously presented). Even theoretical, the proposed control strategy offers a perspective on the properties of a possible magnetic configuration with monotone q-profile

References and publications:

[1] **Gruzinov I., Diamond P. H., Rosenbluth M. N**, "*Sandpiles with Bistable Automata Rules: Toward a Minimal Model of Pedestal Formation and Structure",* Phys. Rev. Letters, vol. 89, 255001(2002)

[2] **Steinbrecher G., Weyssow B.**, "*Generalized Randomly Amplified Linear System Driven by Gaussian Noise. Extreme Heavy Tail and Algebraic Correlation Decay in Plasma Turbulence"*. Physical Review Letters Vol. 92, 125003 (March 2004).

[3] **Rhodes T. L. et al.**, "*Experimental evidence for self-organized criticality in tokamak plasma turbulence"*, Phys. Lett. A 253, 181, (1999).

[4] **Gilmore M., Yu C.X., Rhodes T. L., Peebles W. A.**, "*Investigation of rescaled range analysis, the Hurst exponent, and long time correlations in plasma turbulence"*, Physics of Plasmas, 9, 1312-1317 (2002).

[5] **Weyssow B., Steinbrecher G.**, "*Short range equilibrium models and first exit time statistics of selfsimilar processes"*, Annals of the University of Craiova, (Physics AUC) vol. 15, (2005), (in print).

[6] **Steinbrecher G.**, "*Optimized multidimensional generators for fractional Brownian motion",* Annals of the University of Craiova, (Physics AUC) vol. 14, 127-132, (2004).

[7] **Pometescu N.,** *"Radial and poloidal particle and energy flux of the ions (majority species) in turbulent plasmas in the presence of the ICRH",* International Working Session of the B.F.R.Fusion Group (Belgian-French-Romanian Fusion Group), University of Craiova, Romania, September 2004. (http://membres.lycos.fr/fusionbfr/index00f.html)

[8] **Pometescu N.**, *"Ion radial transport induced by ICRF waves in tokamak non-turbulent plasma*", Annals of the University of Craiova, (Physics AUC), 14 (2004), pg.1-16.

[9] **Balescu R., Petrisor I. and Negrea M.**, "*Anisotropic electrostatic turbulence and zonal flow generation",* (submitted to Plasma Phys. Control. Fusion, (2004)).

[10] **Petrisor I., Negrea M. and Weyssow B.**, "*On the electron diffusion in a sheared unperturbed magnetic field and an electrostatic stochastic field"* (submitted to Plasma Phys. Control. Fusion, (2004)

[11] **Pommois P., Veltri P., Zimbardo G.**, "*Kubo number and magnetic field line diffusion coefficient for anisotropic magnetic turbulence",* Phys.Rev. E, 63, 066405, (2001)

[12] **Horton W., Choi D.-I., Yushmanov P. N. and Parail V. V.,** "*Electron diffusion in tokamaks due to electromagnetic fluctuations"* Plasma Phys. Control. Fusion 29 (1987) 901

[13] **Negrea M., Petrisor I., Balescu R.**, *Intrinsic trapping of stochastic sheared magnetic field lines*, Phys.Rev. E, 70, 046409, (2004)

[14] **Negrea M., Petrisor I.,** "*On the stochastic anisotropic sheared magnetic field lines diffusion"*, Annals of the University of Craiova, (Physics AUC), vol.14, 32-46 (2004)

[15] **Petrisor E., Misguich J. H., Constantinescu D.**, "*Reconnection in a global model of Poincare map describing the dynamics of magnetic field lines in a reversed shear tokanak",* Chaos, Solitons and Fractals 18 (2003), 1085-1099

[16] **Constantinescu D., Constantinescu R.**,*"Transport Barriers and Diffusion Phenomena for the magnetic field lines in tokamak"*, Physica Scripta 71 (2005), (in print)

[17] **Constantinescu D.,** "*Non-linear dynamics and the structure of the magnetic field in tokamak",* Rom. J. Phys. 50**,** no. 3-4 (2005), (in print)

[18] **Constantinescu D., Weyssow B.**, "*Hidden geometrical aspects in the dynamics of nontwist maps"*, Annals of the University of Craiova, (Physics AUC) 14 (2004), pp 150-161

[19] **Constantinescu D., Weyssow B.,** "*Order and Chaos in some Hamiltonian Systems of interest in Plasma Physics"*, Conference Verhulst 200, Ecole Militaire Royale, Bruxelles, September 16-18, 2004, accepted for publication in Proceedings

[20] **Constantinescu D., Misguich J. H., Petrisor E., Pavlenko I.,** "*Internal Transport Barriers in some Hamiltonian systems modelling the magnetic lines dynamics in tokamak"*, 2005 J. Phys.: Conf. Ser. **7** 233-238.